2.1

AUTONOMOUS 1ST ORDER ODE

$$\frac{dy}{dx} = f(y)$$

↳ DERIVATIVE IS EXPRESSED IN TERMS OF ONLY THE DEPENDENT VARIABLE

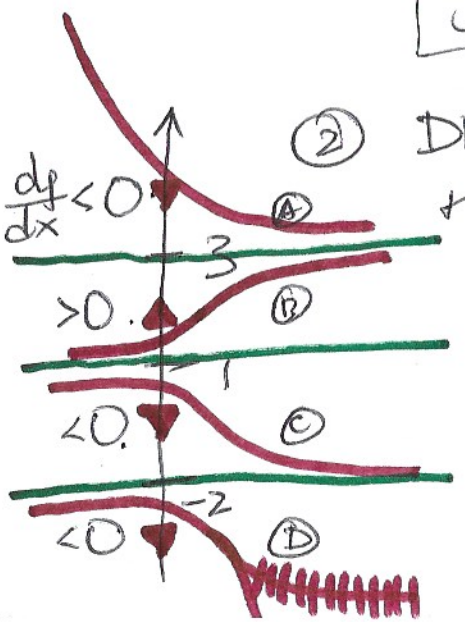
eg. $\frac{dy}{dx} = (y-1)(y+2)^2(3-y)$

① FIND EQ SOL'N'S IE. CONSTANT SOL'NS $y = C$, $y' = 0$

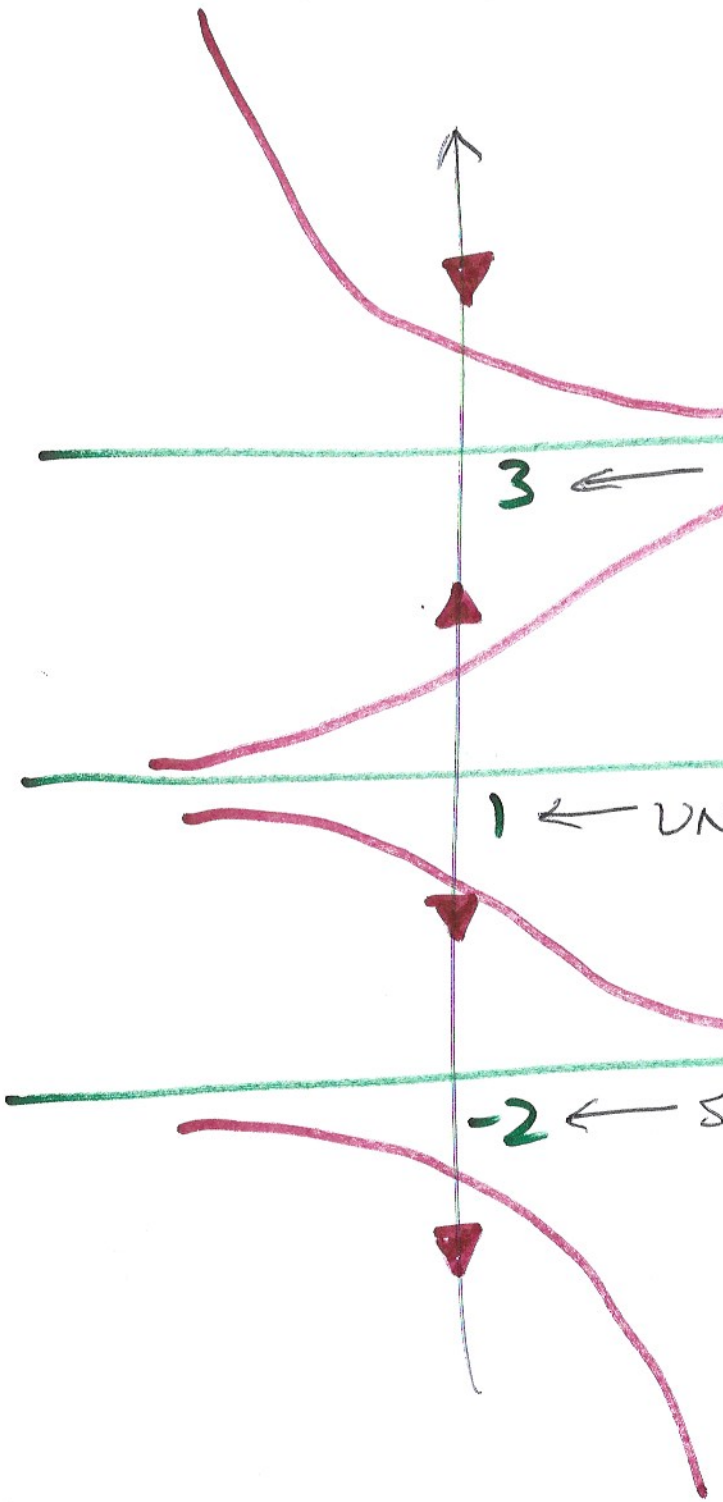
$$0 = (y-1)(y+2)^2(3-y)$$

$$\boxed{y = 1, -2, 3}$$

② DRAW NUMBER LINE DIVIDED BY EQ. SOL'NS (SUB-) + ANALYZE SIGN OF $\frac{dy}{dx}$ IN EACH SUBINTERVAL



(A)	$y > 3$	$(y-1) > 0$	$(y+2)^2 > 0$	$(3-y) < 0$	$\frac{dy}{dx} < 0$
(B)	$1 < y < 3$	> 0	> 0	> 0	$\frac{dy}{dx} > 0$
(C)	$-2 < y < 1$	< 0	> 0	> 0	$\frac{dy}{dx} < 0$
(D)	$y < -2$	< 0	> 0	> 0	$\frac{dy}{dx} < 0$



$\frac{dy}{dx} = (y-1)(y+2)^2(3-y)$

IF $y(-4) = 2$ THEN $\lim_{x\to\infty} y(x) = 3$

IF $y(5) = 0$ THEN $\lim_{x\to\infty} y(x) = -2$

IF $y(0) = 1$ THEN $\lim_{x\to\infty} y(x) = 1$

EQ SOL'N

IF $y(0) = 1.00002$ THEN $\lim_{x\to\infty} y(x) = 3$

IF $y(0) = 0.99998$ THEN $\lim_{x\to\infty} y(x) = -2$

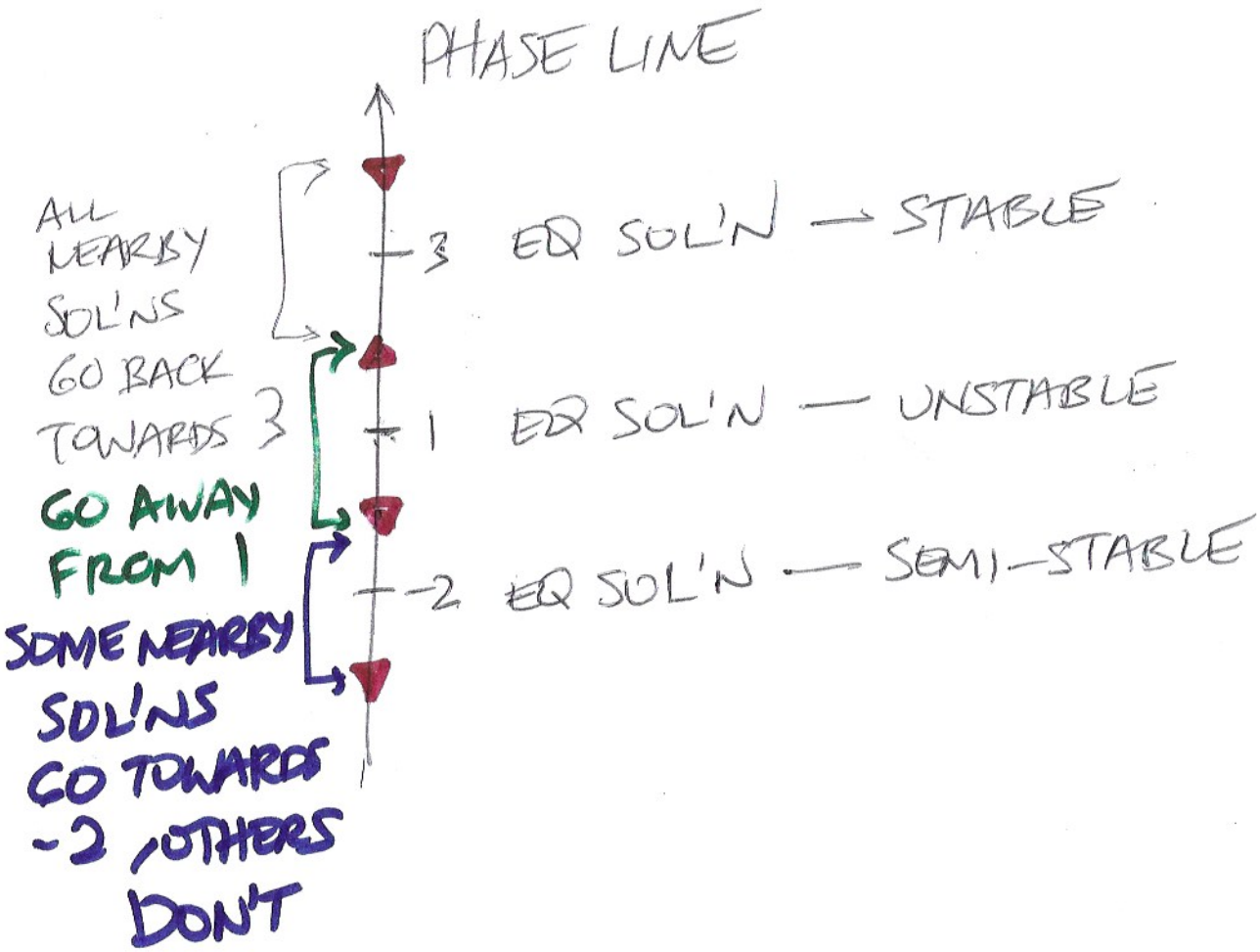
EQ SOL'N

EQ SOL'N

IF $y(5) = 3$ THEN $\lim_{x\to\infty} y(x) = 3$

IF $y(5) = 3.0002$ THEN $\lim_{x\to\infty} y(x) = 3$

IF $y(5) = 2.99998$ THEN $\lim_{x\to\infty} y(x) = 3$


PHASE LINE
ALL NEARBY SOL'NS GO BACK TOWARDS 3
3
EQ SOL'N — STABLE
1
EQ SOL'N — UNSTABLE
GO AWAY FROM 1
2
EQ SOL'N — SEMI-STABLE
SOME NEARBY SOL'NS GO TOWARDS -2, OTHERS DON'T

2.6 EULER'S METHOD

FROM MATH 1A:

LINEAR APPROXIMATION OF $f(x)$ NEAR $x=a$

$$f(x) \approx f(a) + f'(a)(x-a)$$

GIVEN $\frac{dy}{dx} = 2 - \frac{y}{x}$, $y(1) = 3$

NOTE: $y'(1) = \frac{dy}{dx}\Big|_{x=1, y=3} = 2 - \frac{3}{1} = -1$

FOR $x \approx 1$, $y(x) \approx y(1) + y'(1)(x-1)$

$y(x) \approx 3 - 1(x-1)$

$y(x) \approx 4 - x$

eg. $y(2) \approx 4 - 2 = 2$ **BAD ESTIMATE**

Δx TOO LARGE

ACTUALLY, THE SOL'N IS $y = x + \frac{2}{x}$ ✓
OF IVP

$\frac{dy}{dx} = 1 - \frac{2}{x^2}$ AND $y(1) = 1 + \frac{2}{1} = 3$

$$2 - \frac{y}{x} = 2 - \frac{x + \frac{2}{x}}{x}$$

$$= 2 - 1 - \frac{2}{x^2} = 1 - \frac{2}{x^2}$$

AND $y(2) = 2 + \frac{2}{2} = 3$

APPROXIMATE $y(1.1)$

IE. $\Delta x = 0.1$

ESTIMATE $y(1+0.1) = y(1.1) \approx y(1) + y'(1)(1.1-1)$

$y(1.1) \approx 3 + (2-\frac{3}{1})(0.1) = 2.9$

$y(1.2) \approx y(1.1) + y'(1.1)(1.2-1.1)$

$y(1.2) \approx 2.9 + (2-\frac{2.9}{1.1})(0.1) =$ _____

$y(1.3) \approx y(1.2) + y'(1.2)(1.3-1.2)$

⋮

$y(2)$

ACTUALLY

$y = x + \frac{2}{x}$

$y(1.1) = 1.1 + \frac{2}{1.1} \approx 2.9\overline{18}$